Kafkas Universitesi Veteriner Fakultesi Dergisi
ISSN: 1300-6045 e-ISSN: 1309-2251
Journal Home-Page: http://vetdergikafkas.org

Kafkas Univ Vet Fak Derg
27 (5): 547-557, 2021
DOI: 10.9775/kvfd.2020.25171

Research Article

Lactation Milk Yield Prediction with Possibilistic Logistic Regression Analysis

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Article ID: KVFD-2020-25171 Received: 22.11.2020 Accepted: 24.08.2021 Published Online: 24.08.2021

Abstract

The logistic regression is a popular method to model the probability of a categorical outcome given as a dependent variable. However, the possibilistic logistic regression can be preferred instead of classical logistic regression when the dependent variable has uncertainity. The aim of this study is to use the possibilistic logistic regression on animal husbandry examining the theoretical foundations of the method based on fuzzy logic approach. A total of 90 cows were enrolled in the study and the average milk yield in 305 days was predicted by animal's weight, breed of the animal, age in lactation, number of milkings per day and the milking seasons of cows belonging to different breeds. The Mean Degree of Memberships (MDM) and the Mean of Squared Error (MSE) indices were calculated to decide the goodness of fit of the model. The index values were found as MDM=0.896 and MSE=4.871, respectively. It was shown that the model is fit and is succesfull to predict the average milk yield. It can be concluded that the model can provide the businesses on lactation milk yield production an efficient and accurate prediction results.

Keywords: *Fuzzy logistic regression, Lactation milk yield, Possibilistic odds, Minimization, Goodness-of-fit criteria*

Possibilistic Lojistik Regresyon Analizi İle Laktasyon Süt Verimi Tahmini

Öz

Lojistik regresyon analizi, bağımlı değişken olarak verilen özelliğin kategorilerini tahmin etmek için kullanılır. Ancak, bağımlı değişken belirsiz olduğunda klasik lojistik rgeresyon yerine posibilistik lojistik regresyon yöntemi tercih edilebilir. Bu çalışmanın amacı, süt sığırcılığında teorik altyapısı ile birlikte bulanık mantık yaklaşımı temelli posibilistik lojistik regresyon yöntemini kullanmaktır. Çalışmaya toplam 90 inekten elde edilen bilgiler dahil edildi ve hayvanın ağırlığı, hayvanın ırkı, hayvanın laktasyon yaşı, günlük sağım sayısı ve sağım mevsimi bilgileri kullanılarak ortalama süt verimi tahmin edildi. Modelin yeterliliğine karar verebilmek için ortalama üyelik derecesi (MDM) ve hata kareler ortalaması (MSE) indeks değerleri hesaplandı. İndeks değerleri sırasıyla MDM=0.896 ve MSE=4.871 olarak hesaplandı. Bu değerlere göre modelin uyumunun iyi olduğuna karar verildi. Bulgular, modelin laktasyon süt verimini tahminlemede etkin ve güvenilir sonuçlara sahip olduğunu göstermektedir.

Anahtar sözcükler: *Bulanık lojistik regresyon, Laktasyon süt verimi, Minimizasyon, Olabilirlik oranı, Uyum kriterleri*

Introduction

Classical set theory precisely determines the boundaries of sets and the properties of the elements belonging to the set. Each element in the set belongs to a set or not. Partial membership is never allowed. The boundaries of any set to be formed in natural life and the general characteristics of the elements that will form this set cannot always be determined precisely. Because of the uncertainties in the experiences, feelings and thoughts of the experts, their decisions contain an ambiguous language [1]. The question of which criteria and methods would be more accurate to use for more accurate estimation of the yield values of animals and for reliable diagnostic procedures causes disagreements among experts and leads to doubtful decisions. For example, experts who want to classify farm animals according to their milk, meat yields use linguistic expressions such as "low milk yield", "high meat yield", "sick or not sick". These linguistic expressions represent uncertainties belonging to experts [2,3]. It is not

How to cite this article?

Topuz D: Lactation milk yield prediction with possibilistic logistic regression analysis. *Kafkas Univ Vet Fak Derg,* 27 (5): 547-557, 2021.
DOI: 10.9775/kvfd.2020.25171

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a correct choice to apply classical set theory in situations where the knowledge related to the experiences, feelings and thoughts of expert are valid [4]. Because these situations can be very difficult to model since they contain uncertainty. In addition, many scientists state that the source of the uncertainties is the inability to provide the sensitive devices required for measurement or the measurement of erroneous efficiency values due to the use of devices developed according to different criteria or misdiagnosis. In these cases, estimates, classifications and diagnosis processes become questionable because the assumptions required for the application of classical statistical methods cannot be provided [2]. Various theories are used to measure and evaluate uncertainty. Among those theories, probability theory and statistical methods are the preferred methods to model uncertainty [5]. However, many uncertainties encountered in daily life cannot be explained with randomness. Probability theory and statistical methods used to express uncertainties numerically may be inadequate in measuring nonrandom uncertainties [6]. These situations have led researchers to the idea that it is necessary to combine fuzzy sets and statistical theories. Therefore, fuzzy methods based on Zadeh's fuzzy set theory play a very important role in expressing qualitative expressions in human thought numerically in order to make more valid and reliable analyzes [1,7]. Fuzzy logistic regression analysis is one of the results of this combination and is used in cases where the assumptions of the classical logistic regression analysis method cannot be achieved. It can also be used if there is naturally uncertainty in observation values or relationships [8]. Since the approach takes into account "probability" rather than "probabilistic" errors, the error terms are distributed into fuzzy coefficients [9,10]. Studies conducted with fuzzy regression analysis approach are generally based on the applications of linear models. There are very few studies on nonlinear models.

The aim of this study was conducted to examine the theoretical foundations of possibilistic logistic regression analysis technique based on fuzzy logic approach and its application in animal husbandry, which can provide solutions to fuzzy situations. In order to estimate its model parameters, the applicability of the Diamond's Possibilistic method in animal husbandry proposed by Pourahmad et al was discussed for in this study.

Material and Methods

Ethical Statement

Data collection with the animal care and breeding practices from enterprise were used in this study in compatible with animal welfare rules stated in Article 9 in government law in Turkey (No.5996).

Materials

The material of the study consisted of 2005 milk yield records of 90 randomly selected cows from 220 cows of different breeds (Holstein Friesian, Brunette, South anatolian red, Cross breed) raised in a private farm. In order to estimate the likelihood values of average milk yield (Y_i) (kg) values in a lactation period with minimum error, values of independent variables such as the milk yield of cows in lactation (X_1), weight (X_2), breed (X_3), age in lactation (X_4), number of milkings per day (X_5) and milking season (X_6) were used as material. In addition, the Mean Degree of Memberships (MDM) and Mean of Squares Errors (MSE) indices were used to decide the adequacy of the model created.

Possibilistic Logistic Regression Analysis Model

Logistic regression model is the most commonly used regression model in cases where the dependent variable is categorical [11]. Logistic regression analysis is preferred when the relevant dependent variable consists of categories such as "low efficiency - high efficiency", "sick - not sick". Fuzzy logistic regression analysis approach is a regression method based on fuzzy set theory, and it is a fuzzy approach used to analyze the uncertainties in the natural structure of the data which belongs to the dependent variable [11,12]. The approach is a regression method based on fuzzy set theory used in cases where classical logistic regression analysis assumptions cannot be fulfilled or the data is fuzzy due to its nature [8]. It can also be used in situations where observation values or relationships between variables are uncertain [7,13-15]. The linear regression is not applicable to some situations such as when there is a small data set, vagueness in the relationship between the predictors and response variables, and inaccuracy or distortion introduced by linearization. Therefore, fuzzy regression is generally performed to complement those situations and problems [6]. Logistic regression modeling is a nonlinear statistical method used to model a categorical response variable based on some covariates [1]. In fuzzy field, fuzzy logistic regression for binary base response is also defined. Studies on this topic can be categorized into possibilistic methods and distance-based methods. A common viewpoint which is applied by Nagar and Srivastava [4] simultaneously, used a possibilistic-based approach to investigate a certain fuzzy logistic regression model. In fact, they used this approach to predict the oral cancer based on some real data sets.

Since the estimated values of the dependent variable are calculated by probability, the equations belonging to the fuzzy logistic regression model showing the possibilistic value are as follows [4,9,11,13-16];

$$\widetilde{W}_j = \ln\left\{\frac{\widetilde{\mu}_i}{[1-\widetilde{\mu}_i]}\right\} = \widetilde{A}_0 + \widetilde{A}_1 x_{i1} + \widetilde{A}_2 x_{i2} + \widetilde{A}_3 x_{i3} + \cdots + \widetilde{A}_{p-1} x_{i(p-1)} \quad (1)$$

Here; It is a data set which is $X_i = [x_{i1}, x_{i2}, .., x_{i(p-1)}]\ i = 1, \ldots, n$ and it is the observation vector of independent variables consisting of precise values such as milk yield, weight, race, age in lactation, daily milking number and milking season of i^{th} cow. Each independent variable observation is expressed as $x \in X$

(i: 1,2,3,..,p (p, number of independent variables). $\tilde{\mu}_i$: is the value indicating that the i[th] unit can have the desired property for the relevant dependent variable as $\tilde{\mu}_i = \mathrm{Poss}(Y_i = 1)$ [6,12,17,18].

$\tilde{A}_i = [\tilde{A}_0 \text{ ve } \tilde{A}_1, \tilde{A}_2, \dots \tilde{A}_i \dots, \tilde{A}_{p-1}]$ is the coefficient values of the independent variables in the function, represented by $\tilde{A}_i = [a_i^c, a_i^s]$ [2,19,20]. a_i^c: is the mode value representing the center of the fuzzy coefficients and is in the form of $a_i^c = [a_0^c, a_1^c, \dots, a_n^c]$. a_i^s: is the value showing the spread of fuzzy coefficients and it is in the form of $a_i^s = [a_0^s, a_1^s, \dots, a_n^s]$. Hence each coefficient is defined by $\tilde{A}_0 = [a_0^c, a_0^s]$, $\tilde{A}_1 = [a_1^c, a_1^s], \dots, \tilde{A}_6 = [a_6^c, a_6^s]$ [3,20]. $\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}$: shows the probability that the i[th] case will have the property considered for each fuzzy case, and is called possibilistik odss [9,15]. $\tilde{W}_j = \ln\left\{\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}\right\}$: estimated fuzzy output value [4,11].

Estimation of Fuzzy Coefficients

The approach suggested by Pourahmad et al.[14] based on Diamond's likelihood (possibilistic) approach was used [3,21-23]. The main purpose is to minimize the total uncertainty of the model by minimizing the total spread of fuzzy coefficients. $\tilde{A}_i = (a_j^c, s_j^L, s_j^R)$: $s_j^L = s_j^R = s_j$ $j = 0,1,\dots,p-1$ the coefficient values are assumed as triangular fuzzy numbers because they represent measurement errors and general uncertainties. Fuzzy output $\tilde{W}_j(j = 1,\dots,m)$ values estimated by equation (1) according to fuzzy arithmetic operations are also modeled as in equation (2) because they are fuzzy numbers with symmetrical triangular properties [19],

$$\tilde{W}_j = f(x, \tilde{A}) = [f_i^c(x), f_i^s(x)] = [\,f_i^c(x), f_{is}^L(x),\ f_{is}^R(x)]_T \quad (2)$$

Here, $f_i^s(x)$: is the spread of the fuzzy logistic regression approach and is as $f_i^s(x) = [a_0^s + a_1^s x_{i1} + a_2^s x_{i2} +, \dots, + a_n^s x_{in}]$. $f_i^c(x)$ is the mode value of the fuzzy logistic regression approach as in equation (3) [24].

$$f_i^c(x) = a_0^c + a_1^c X_{i1} + \dots + a_{P-1}^c X_{in},\ F_{is}^L(x) = s_0^L + s_1^L X_{i1} + \dots + s_{p-1}^L X_{in},$$
$$f_{is}^R(x) = s_0^R + s_1^R X_{i1} + \dots + s_{P-1}^R X_{in} \quad (3)$$

The $\tilde{W}_j$ membership function of the dependent variable, estimated by the fuzzy logistic regression analysis method, can be shown as in equation (4);

$$\tilde{W}_j(\tilde{Y}_j) = \begin{cases} 1 - \frac{f_i^c(x) - \tilde{Y}_j}{f_{is}^L(x)},\ f_i^c(x) - f_{is}^L(x) \le \tilde{Y}_j \le f_i^c(x), \\ 1 - \frac{\tilde{Y}_j - f_i^c(x)}{f_{is}^R(x)},\ f_i^c(x) < \tilde{Y}_j \le f_i^c(x) + f_{is}^R(x), \\ 0 \qquad \text{otherwise} \end{cases} \quad (4)$$

If the $\tilde{W}_j = f(x, \tilde{A}) = [f_i^c(x), f_i^s(x)]$ $\tilde{A}_i = [a_i^c, a_i^s]$: $s_i^L = s_i^R = s_i$ $j = 0,1,\dots,p-1$ values are equal to each other. This state is expressed as an [15,16,19].

$$\tilde{W}_j = [f_i^c(x), f_i^S(x)] = f_{is}^L(x) = f_{is}^R(x) = f_i^S(x) \ \ (i = 1,2,3,\dots,m) \quad (5)$$

These values will be obtained as a fuzzy number with symmetric triangular property equal to each other. So, after estimating the model coeficients, we can determine the membership function of the possibilistic odds, $f(\tilde{W}_j) = \exp[\tilde{W}_j(x)], x > 0$ as follows; membership function is created as in equation (6) [4,11,15,16].

$$\exp[\tilde{W}_j(x)] = \tilde{W}_j(\ln(x)) = \begin{cases} 1 - \frac{f_i^c(x) - \ln(x)}{f_{is}^L(x)},\ f_i^c(x) - f_i^L(x) \le \ln(x) \le f_i^c(x), \\ 1 - \frac{\ln(x) - f_i^c(x)}{f_{is}^R(x)},\ f_i^c(x) < \ln(x) \le f_i^c(x) + f_i^R(x). \end{cases} \quad (6)$$

These assumptions are as follows:

1-The real lactation milk yield value $(\tilde{Y}_j)$ of each cow should be estimated from the $(\tilde{W}_i)$ value estimated with the fuzzy approach, with a membership degree of the least turbidity tolerance coefficient (h) value$\left(\tilde{W}_j(\tilde{Y}_j) \ge h\right)$ [13]. Here $\tilde{Y}_j$: is the observed value of the dependent (response) variable, which is a definite number, and is as

$\tilde{Y}_j = \ln\left\{\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}\right\}, h \in (0.1)$ [11,25].

2- The objective function minimizing the $(\tilde{W}_i)$ spread of the fuzzy output values to be estimated is as in equation (7) [14,15,24];

$$Z = [m(s_0^L + s_o^R)] + \sum_{j=1}^{p-1}\left[(s_j^L + s_j^R)\sum_{i=1}^{n} xij\right] \quad (7)$$

Here, n: is the number of observations regarding the dependent variable. j: the number of independent variable. x_{ij}: i[th] observation value of the j[th] independent variable. The number of observations n determines the number of constraints because a range is estimated for $\tilde{Y}_j$ by approaching from the left and right. The constraint number for each estimated $\tilde{W}_i$ value should be 2xn [21]. Using Equation (6), the constraint limitation of each situation $\tilde{W}_j(\tilde{Y}_j) \ge h, j = 1,2,\dots,n$ is as in equations (8) and (9) [5,15]:

$$1 - \frac{f_i^c(x) - \tilde{Y}_j}{f_{is}^L(x)} \ge h \Rightarrow (1-h)f_{is}^L(x) - f_i^c(x) - \ge \tilde{Y}_j \qquad i = 1,2,3,\dots,n$$
$$1 - \frac{\tilde{Y}_j - f_i^c(x)}{f_{is}^R(x)} \ge h \Rightarrow (1-h)f_{is}^R(x) + f_i^c(x) \ge \tilde{Y}_j \qquad i = 1,2,3,\dots,n \quad (8)$$

By rearranging (8) Equality, we can create constraints [12] for sample data sets as in equation (9)

$$(1-h)s_o^L + (1-h)\sum_{j=1}^{p-1}(s_i^L x_{ij}) - a_o^c - \sum_{j=1}^{p-1}(a_i^c x_{ij}) \ge -\tilde{Y}_j \quad \forall i, i = 1,2,\dots,n$$
$$(1-h)s_o^R + (1-h)\sum_{j=1}^{p-1}(s_j^R x_{ij}) + a_o^c + \sum_{j=1}^{p-1}(a_i^c x_{ij}) \ge \tilde{Y}_j \quad \forall i, i = 1,2,\dots,n \quad (9)$$

The first constraint is for calculating the mean and left and right spreads of each coefficient, while the second constraint is for minimizing the objective function based on linear programming (Equation 8) [5,7,26,27].

Goodness of Fit Test Criterion for Fuzzy Regression Models

Fuzzy logistic regression attempts to model and predict the possibility of success based on fuzzy covariates. To appraise the goodness of fit of the fuzzy logistic regression, we introduce some criteria. These are "Mean degree of membership (possibilistic odds) (MDM), "Mean of Squares Errors (MSE) [16].

Mean Degree of Memberships (MDM)

The average degree of membership is a criterion similar to the coefficient of expression (R^2), which indicates how much of the observed change in the dependent variable is explained by the observed change in the independent variables [22]. With the help of an average membership level of 10;

$$\mathrm{MDM} = \frac{1}{m}\sum_{i=1}^{n} \widetilde{W}_j(\widetilde{Y}_j) = \frac{1}{m}\sum_{i=1}^{n} \exp\left[\widetilde{W}_j\left(\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}\right)\right] \quad (10)$$

can be calculated [27]. Here; $\widetilde{Y}_j$: is the observed value of the dependent variable, which is an absolute bivalent number for the j^{th} case. $\widetilde{W}_j$: is the natural logarithm of the odds of possibilistic estimated as a fuzzy number. $\exp\left[\widehat{W}_i\left(\frac{\mu_i}{[1-\mu_i]}\right)\right] = \frac{\mu_i}{[1-\mu_i]}$: i^{th} is the estimated value of the possibility of the odds which belongs to the i^{th} observation. By the help of Equation (11);

$$\exp\left[\widetilde{W}_j\left(\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}\right)\right] = 1 - \frac{\left(\widetilde{Y}_j - f_i^c(x)\right)}{f_i^s(x)} \ \text{veya}\ 1 - \frac{\left(f_i^c(x) - \widetilde{Y}_j\right)}{f_i^s(x)} \quad (11)$$

can be calculated. It is desirable that the average of the calculated membership degrees is close to 1. Average membership degree takes values between 0 and 1 [16,26,27].

Mean Square Error Test Criteria (MSE)

It is an index used to evaluate the goodness of fit. The closer the predicted value is to the observed value, the stronger the model's power to predict real situations [17]. The calculation formula is as Equation (12) [9,28];

$$\mathrm{MSE} = \frac{1}{m}\sum_{i=1}^{n}\left[\mathrm{defCoG}\left\{\exp\left[\widetilde{W}_j\right]\right\} - \left(\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}\right)\right]^2 \quad (12)$$

Here; $\mathrm{defCoG}\left\{\exp\left[\widetilde{W}_j\right]\right\}$, $\exp\left[\widetilde{W}_j\left(\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}\right)\right]$ is the estimated value of the possibilistic odds for defuzzification. Since the estimated fuzzy values will have to be converted to numerical (exact, net) values, the clarification process using the center of Gravity defuzzication method, which is one of the clarification methods, is determined by equation (13) and,

$$\mathrm{def\,CoG}\left\{\exp\left[\widetilde{W}_j\right]\right\} = \frac{\int_{\exp(f_i^c(x)-f_i^s(x))}^{\exp(f_i^c(x))} x\left(1 - \frac{f_i^c(x) - \ln(x)}{f_i^s(x)}\right)dx + \int_{\exp(f_i^c(x))}^{\exp((f_i^c(x)+f_i^s(x)))} x\left(1 - \frac{\ln(x) - f_i^c(x)}{f_i^s(x)}\right)dx}{\int_{\exp(f_i^c(x)-f_i^s(x))}^{\exp((f_i^c(x)))} x\left(1 - \frac{f_i^c(x) - \ln(x)}{f_i^s(x)}\right)dx + \int_{\exp(f_i^c(x))}^{\exp((f_i^c(x)+f_i^s(x)))} x\left(1 - \frac{\ln(x) - f_i^c(x)}{f_i^s(x)}\right)dx} \quad (13)$$

can be calculated [15,28].

Statistical Analysis

The statistical analyses were performed by Microsoft Office EXCEL 2016 (R) and LINGO 16.0 softwares. The MDM and MSE values were calculated to decide the adequecy of the model.

RESULTS

Step 1. Experts and producers may be suspicious of the measurements made because sensitive devices required for the measurement of average milk yield values in a lactation period cannot be provided or due to measurement errors. In such cases, they cannot represent the average milk yield value of each animal by one of the two dependent response categories. In other words, since the observed values of the dependent variable do not represent a certain situation, it cannot be stated that they belong to any category as 0 or 1. Therefore, due to the uncertainty in the dependent variable, the probability that the i^{th} animal belongs to the category 1 ($E(Y) = P(Y = 1) = p$) and its odds ratio $\left\{\frac{p_i}{[1-p_i]}\right\}$ cannot be calculated [13,29].

For this purpose, independent variables were determined as milk yield of cows in lactation (X_1), weight (X_2), race (X_3), age at lactation (X_4), daily milking number (X_5) and milking season (X_6). The dependent variable is whether the average milk yield (Y_i) (kg) values in a lactation period have normal yield values according to the mentioned independent variables. Expert opinion was taken to determine the likelihood of lactation milk yield values. In order to determine the likelihood of lactation milk yield values, we defined a measure such as $\tilde{\mu}_i \in$ { No high milk yielding cow=0, High milk yielding cow=1}, as the degree of consistency (probability measure) of the known characteristic for each turbid state representing the average milk yield (Y_i) (kg) values (LSV) in a lactation period (LSV <3000 kg = 0, LSV≥3000 kg = 1). According to the defined measure $\tilde{\mu}_i$: is the probability of whether the animal is a low or high milk yield one or not, instead of 0 and 1, which replace the observed value of the dependent variable and the degree of membership indicating the likelihood of belonging to any category in the $\mu_i \in R: 0 \leq \tilde{\mu}_i \leq 1$ [12,16,17,26] range i =1,2,.., 90. The complement of each situation is $1 - \tilde{\mu}_i \in R: 0 \leq 1 - \tilde{\mu}_i \leq 1$. The $\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}$ value indicating the ratio of the probability of having a certain property of the related fuzzy observation and not being possible is called "possibilistic odds" [4,23].

In order to apply the fuzzy logistic regression analysis method, the possibility of the i^{th} case to have this related property due to the uncertainty in the dependent variable is expressed as $\tilde{\mu}_i = \mathrm{Poss}(Y_i = 1)$[4,10,12]. We calculated the probability ratios by applying logarithmic transformations for the values of the binary dependent variables according to different significance levels.

Within the same logic, we calculated the measurement value

Table 1. The number of fuzzy triangle $\widetilde{\mu}_j$ and its corresponding $\widetilde{Y}_j$ transformations				
No	**Linguistic Expression of the Fuzzy Binary Dependent Variable (Y_i)**	$\widetilde{\mu}_i$	$\frac{\widetilde{\mu}_i}{[1-\widetilde{\mu}_i]}$	$\widetilde{Y}_j = \ln\left\{\frac{\widetilde{\mu}_i}{[1-\widetilde{\mu}_i]}\right\}$
	0	(0.01; 0.05)	(0.01; 0.05)	(-4.605; -2.995)
	1	(0.95; 0.99)	(19; 99)	(2.944; 4.595)
1	1	0.283	0.395	-0.9281
2	1	0.142	0.166	-1.7938
.	.	.	.	.
89	1	0.584	1.404	0.3392
90	0	0.029	0.030	-3.5110
** Not having a high milk yield cow = 0, High milk yielding cow = 1*				

corresponding to $\widetilde{\mu}_i$ and the probability values of 90 cows as a triangular fuzzy number ($\widetilde{Y}_j$) with the symmetric triangular membership function, as in *Table 1*.

We used these calculated fuzzy values as output observation values.

Step 2. In order to minimize the uncertainties of independent variables affecting the average milk yield in a lactation period at the level of h=0.5, constraint values from $\widetilde{Y}_1 = \log_e\left\{\frac{0.2833}{[1-0.2833]}\right\} = -0.9281$ to $\widetilde{Y}_{90} = \log_e\left\{\frac{0.0290}{[1-0.0290]}\right\} = -3.5110$ were calculated and 180 (90*2) constraint matrices were created as in equation (14):

$$\text{MIN} = 90 * a_0^s + 46903 * a_1^s + 7799 * a_2^s + 28 * a_3^s + 1178 * a_4^s + 51 * a_5^s + 9 * a_6^s$$

$$\widetilde{Y}_1 = \begin{bmatrix} -0.5 * a_0^s - 3720 * a_1^s - 563 * a_2^s - 0 * a_3^s - 78 * a_4^s - 3 * a_5^s - 1 * a_6^s + a_o^c + 3720 * a_1^c + 563 * a_2^c + 0a_3^c + 78 * a_4^c + 3 * a_5^c + 1 * a_6^c <= -0.928; \\ 0.5 * a_0^s + 3720 * a_1^s + 563 * a_2^s + 0 * a_3^s + 78 * a_4^s + 3 * a_5^s + 1 * a_6^s + a_o^c + 3720 * a_1^c + 563 * a_2^c + 0 * a_3^c + 78 * a_4^c + 3 * a_5^c + 1 * a_6^c >= -0.928; \end{bmatrix}$$

$$. \qquad\qquad . \tag{14}$$

$$\widetilde{Y}_{90} = \begin{bmatrix} -0.5 * a_0^s - 786 * a_1^s - 176 * a_2^s - 3 * a_3^s - 41 * a_4^s - 2 * a_5^s - 0 * a_6^s + a_o^c + 786 * a_1^c + 176 * a_2^c + 3 * a_3^c + 41 * a_4^c + 2 * a_5^c + 0 * a_6^c <= -3.511; \\ 0.5 * a_0^s + 386 * a_1^s + 176 * a_2^s + 3 * a_3^s + 41 * a_4^s + 2 * a_5^s + 0 * a_6^s + a_o^c + 386 * a_1^c + 176 * a_2^c + 3 * a_3^c + 41 * a_4^c + 2 * a_5^c + 0 * a_6^c >= -3.511; \end{bmatrix}$$

@FREE(C1); @FREE(C2); @FREE(C3); @FREE(C4); @FREE(C5); FREE(C6); END

The first six elements in each row of the constraint matrices show the center (a_i^c) values, and the last six elements show the spread (a_i^S) values.

Step 3. The possibilistic model to be created at the h = 0.5 level is

$\widetilde{W}_j = \widetilde{A}_0 + \widetilde{A}_1 * \text{LSV} + \widetilde{A}_2 * \text{HA} + \widetilde{A}_3 * \text{HI} + \widetilde{A}_4 * \text{HLY} + \widetilde{A}_5 * \text{GSS} + \widetilde{A}_6 * \text{SM},$ j = 1,2,…,90

To estimate the $\widetilde{A}_i$, i = 0,1,...,6, coefficients which belong to the model, Diamond's possibilistic method, which is based on the linear programming method, was used [23]. The values in *Table 2* are calculated by using the constraint matrices in the equation (14) created with this logic approach in "Lingo 16.0" software [16,21].

Studies have determined that changing the value of h does not affect the center of the coefficients (a_i^c) but affects the spread (a_i^s) and the value of the objective function (Z) [5].

Step 4. The fuzziness of the model to be created at the level of h = 0.5, considering the total values of the variables and the dispersion values, was calculated as in the Equation (15) [21,28].

Table 2. Coefficient values of fuzzy logistic regression approach					
Variables	$\widetilde{A}_i = (a_i^c; a_i^s)$			**Classical Logistic Regression**	
	$\widetilde{A}_i$	a_i^c	a_i^s	$\widehat{\beta}$	**Std. Error**
Constant	$\widetilde{A}_0$	3.6932	0.000	-0.4225	0.364
LSV(X_1)	$\widetilde{A}_1$	0.0005	0.000	-1.3646	0.859
HA(X_2)	$\widetilde{A}_2$	0.0050	0.000	0.0782	2.078
HI(X_3)	$\widetilde{A}_3$	-1.2546	0.523	1.6721	1.052
HLY(X_4)	$\widetilde{A}_4$	-0.0228	0.069	-0.0272	2.194
GSS(X_5)	$\widetilde{A}_5$	-2.6493	0.000	2.9756	0.157
SM(X_6)	$\widetilde{A}_6$	0.3736	0.000	-1.7321	0.675

LSV: lactation milk yield, HA: animal's weight, HI: breed of the animal (Holstein Friesian = 0, brunette = 1, south anatolian red = 2, cross breed = 3), HLY: the animal's age at lactation (Moon), GSS: number of milkings per day, SM: milking season (winter and autumn = 0, spring and summer = 1)

$$Z = \left[90\, a_0^S + a_1^S \sum_{i=1}^{90} x_{1i} + a_2^S \sum_{i=1}^{90} x_{2i} + a_3^S \sum_{i=1}^{90} x_{3i} + a_4^S \sum_{i=1}^{90} x_{4i} + a_5^S \sum_{i=1}^{90} x_{5i} + a_6^S \sum_{i=1}^{90} x_{6i}\right] \tag{15}$$

$$Z = [90 * 0.0 + 46903 * 0.0 + 7799 * 0.0 + 28 * 0.5235 + 1178 * 0.0693 + 51 * 0.0 + 9 * 0.0]$$

Z_{1X12} = 96.293. This objective function, calculated in 90x14 dimensions, is minimized by limiting it to 180 (90 observations × 2) matrix of coefficients.

Step 5. The best fuzzy logistic regression analysis equation created at Z = 96.293 fuzziness value is calculated as in the Equation (16).

$$\begin{aligned}\widetilde{W}_j &= (3.6932; 0.0) + (0.0005; 0.0) * X_1 + (0.0050; 0.00) * X_2 + (-1.2546; 0.523) * X_3 \\ &+(-0.0228; 0.069) * X_4 + (-2.6493; 0.0) * X_5 + (0.3736; 0.0) * X_6\end{aligned} \tag{16}$$

With Equation (16), it is tried to determine whether each cow is a high milk producing or low milk producing cow with the constraint lines of the independent variable values used. The possibilistic probability values of the lactation milk yield values to be estimated in these unclear situations are a fuzzy number with a symmetrical triangular feature. When we want to calculate the posibilistic odds of ratio of the average milk yield value of the number one cow in a lactation period, the relevant data are applied and calculated in equation (17);

$$\begin{aligned}\widetilde{W}_1 = \left[\ln\left(\frac{\widetilde{\mu}_1}{[1-\widetilde{\mu}_1]}\right)\right] &= (3.6932; 0.0) + (0.0005; 0.0) * 3720 + (0.0050; 0.0) * 563 + (-1.2546; 0.523) * 0 + (-0.0228; 0.069) * 78 \\ &+ (-2.6493; 0.0) * 3 + (0.3736; 0.0) * 1 \\ &= (3.6932; 0.0) + (1.86; 0.0) + (2.815; 0.00) + (0.00; 0.00)_T + (-1.7784; 5.382) + (-7.9479; 0.0) + (0.3736; 0.0) = (-1.9850; 5.382)_T\end{aligned} \tag{17}$$

Here, $\widetilde{W}_1 \cong (-1.98;\ 5.38)_T$ is the natural logarithm of the possibilistic odds of ratio of the average milk yield value of the number one cow in a lactation period and is -1.98.

Step 6. For each cow, calculations were made as in equation (17) and the values in *Table 3* were obtained.

Step 7. In order to calculate the predicted value of the possibilistic odds of ratio calculated for each cow in the sixth step, extension principle in equation (6) is used. For example, the posibilistic odds of ratio of the average amount of milk i lactation period belonging to number one cow ($\exp[\widetilde{W}_1] = \frac{\widetilde{\mu}_1}{[1-\widetilde{\mu}_1]}) = \widetilde{W}_1\left[\ln\frac{x}{1-x}\right]$ is calculated as in equation (18) [20].

Table 3. Some calculated fuzzy statistical values of sample data set			
No	$\widetilde{Y}_j = \ln\left\{\frac{\widetilde{\mu}_i}{[1-\widetilde{\mu}_i]}\right\}$	**Estimated Values with Fuzzy Approach: $\widetilde{W}_j(x)$**	
		$f_i^C(x)$	$f_i^S(x)$
1	-0.928	-1.98	5.38
2	-1.794	-1.52	5.93
.	.	.	.
89	0.339	-0.58	3.83
90	-3.511	-5.03	4.40

$$\exp[\widetilde{W}_1] = \left\{\begin{matrix} 1-\dfrac{-1.98-\ln\left(\frac{x}{1-x}\right)}{5.38}, -7.36 \le \ln\left(\frac{x}{1-x}\right) \le -1.98\ [0.0063 \le x \le 0.138] \\ \\ 1-\dfrac{\ln\left(\frac{x}{1-x}\right)+(-1.98)}{5.38}, -1.98 < \ln\left(\frac{x}{1-x}\right) \le 3.40[0.138 \le x \le 29.96] \end{matrix}\right\} \tag{18}$$

Possibilistik odds of having a high milk yield cow according to the fuzzy observation values of cow number one was calculated as 0.138.

Step 8. Since the calculated $\frac{\widetilde{\mu}_i}{[1-\widetilde{\mu}_i]}$ rate values can take values between 0 and $+\infty$, logit transformation is made by taking the natural logarithms of the odds values (logit is made by taking the natural logarithm of the odds value of an event). As a result of the transformation, the nonlinear logistic regression function is transformed into a linear symmetric function by ensuring that the limits of $\widetilde{Y}_j = \ln\left\{\frac{\widetilde{\mu}_i}{[1-\widetilde{\mu}_i]}\right\}$ values are taken from the range $(0,+\infty)$ to the limit $(-\infty,+\infty)$ [28]. The estimated fuzzy likelihood $(\widehat{P}_i)$ values are calculated by applying transformation to the estimated posibilistic odds ratio values. Because $0.138 = \frac{\widehat{\mu}_1}{[1-\widehat{\mu}_1]}$ for number one cow, It has been calculated as $\widehat{\mu}_1 = 0.138 - 0.138 * \widehat{\mu}_1 \rightarrow \widehat{\mu}_1 + 0.138 * \widehat{\mu}_1 = 0.138 \rightarrow 0.138 * \widehat{\mu}_1 = 0.138 \rightarrow \widehat{\mu}_1 = \frac{0.138}{1.138} =$ 0,121.

This calculated value is the high milk yield likelihood value of number one cow and it was calculated as $\exp[\widetilde{W}_1] = \widetilde{W}_1\left[\ln\frac{x}{1-x}\right] = \widetilde{W}_1(-1.98) = \exp(\widetilde{W}_1(0.395)$. This calculated value is explained with the $\exp(\widetilde{W}_1(0.395)$ =0.121 possibilistic that the number one cow is the normal milk producing cow. And for the cow no 90, it is calculated as $\exp[\widetilde{W}_{90}]$;

$$\exp[\widetilde{W}_{90}] = \left\{\begin{matrix} 1-\dfrac{-5.03-\ln\left(\frac{x}{1-x}\right)}{4.40}, -5.03-4.40 \le \ln\left(\frac{x}{1-x}\right) \le -5.03\ [e^{-5.03-4.40} \le x \le e^{-5.03}] \\ \\ 1-\dfrac{\ln\left(\frac{x}{1-x}\right)+(-5.03)}{4.40}, -5.03 < \ln\left(\frac{x}{1-x}\right) \le -5.03+4.40[e^{-5.03} \le x \le e^{-5.03+4.40}] \end{matrix}\right\} \tag{19}$$

$\exp[\widetilde{W}_{90}] = (-3.51) = \exp(\widetilde{W}_{90}(0.03)$. $\exp[0.395] \ge h = 0.5$ is indicated by the value of tolerance coefficient (h). This calculated value is the predicted value of the possibilistic odds of the cow numbered ninety to be a high milk producing cow. Another important feature of the fuzzy logistic regression analysis approach is that the probability of probability (posibilistic odds) value can be predicted when a new cow other than the cows used is added to the study or business. For example, suppose that the 91th low milk yield cow with an average lactation milk yield of 2850 kg and $x_2 = 630$, $x_3 = 0$, $x_4 = 41$, $x_5 = 2$, $x_6 = 0$ will be included in the enterprise. By applying these values at equation (16), possibilistic odds ratio of average milk yield in a lactation period (Possibilistic odds) is calculated as:

$$\begin{aligned} \widetilde{W}_{91} &= \left[\log_e\left(\frac{\widetilde{\mu}_{91}}{[1-\widetilde{\mu}_{91}]}\right)\right]; (3.6932; 0.0) + (0.0005; 0.0) * 2850 + (0.0050; 0.00) * 630 + \\ &(-1.2546; 0.523) * 0 + (-0.0228; 0.0693) * 41 + (-2.6493; 0.0) * 2 + (0.3736; 0.0) * 0 \\ &= (2.143; 2.866)_T \cong (2.14; 2.87)_T \end{aligned} \tag{20}$$

According to the fuzzy observation values of the newly added cow, the value showing the possibilistic odds of a low milk yield cow can be calculated as follows: $\exp[\widetilde{W}_{91}]$;

$$\exp[\widetilde{W}_{91}] = \left\{ \begin{array}{l} 1 - \dfrac{2.14 - \ln\left(\frac{x}{1-x}\right)}{2.87}, -0.73 \le \ln\left(\dfrac{x}{1-x}\right) \le 2.14\ [0.482 \le x \le 8.49] \\ 1 - \dfrac{\ln\left(\frac{x}{1-x}\right) + (2.14)}{2.87}, 2.14 < \ln\left(\dfrac{x}{1-x}\right) \le 5.01\ \ [8.49 \le x \le 149.90] \end{array} \right\} \quad (21)$$

It is seen that the posibilistic odds value of this newly added cow is calculated as 8.49. This value is the value that indicates the ratio of the probability of cow 91 being a low-yielding animal to the probability of not being low milk yield. With the possibility of 0.894, it can be said that the newly added cow no 91 has low milk yield after performing the relevant reverse conversions. Calculations for each other cow were made in this way and the values in *Table 4* were obtained.

Table 4. *Observed and predicted fuzzy statistical values of sample data set*

No	$\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}$	$\tilde{Y}_j = \ln\left\{\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}\right\}$	Estimated Values with Fuzzy Approach: $\widetilde{W}_j(x)$		The Predicted Values of Probability of Average Milk Yield Values According to the Expansion Principle: $\exp\left[\widetilde{W}_j\left(\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}\right)\right] = \widetilde{W}_j(\tilde{Y}_j)$
			$f_i^C(x)$	$f_i^S(x)$	
1	0.395	-0.928	-1.98	5.38	$\widetilde{W}_1(-0.928) = \exp(\widetilde{W}_1(0.395)$
2	0.166	-1.794	-1.52	5.93	$\widetilde{W}_2(-1.794) = \exp(\widetilde{W}_2(0.166)$
.	.	.	.		. . .
89	1.404	0.339	-0.58	3.83	$\widetilde{W}_{89}(0.339) = \exp(\widetilde{W}_{89}(1.404)$
90	0.030	-3.511	-5.03	4.40	$\widetilde{W}_{90}(-3.511) = \exp(\widetilde{W}_{90}(0.030)$

Step 9. In order to calculate the "Average Degree of Membership (MDM)" value of the fuzzy logistic regression analysis model in Equation (16), the average milk yield value for each cow in a lactation period and the estimated turbid output (mean milk yield) value are used in equations (11) and (12). Membership degrees of the predicted possibilistic odds of average milk yield values in a lactation period for each cow are calculated as:

$$\widetilde{W}_1(x) = \left\{ \begin{array}{l} 1 - \dfrac{-1.98 - \ln\left(\frac{x}{1-x}\right)}{5.38} \\ 1 - \dfrac{\ln\left(\frac{x}{1-x}\right) + (-1.98)}{5.38} \end{array} \right\} \Rightarrow \widetilde{W}_1(x) = 1 - \frac{(-0.93) - (-1.98)}{5.38} = \widetilde{W}_1(-0.93) \cong 0.80 \quad (22)$$

And the values in *Table 5* are obtained.

Table 5. Membership degrees of observed and predicted turbid output values of lactation milk yields

No	$\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}$	$\tilde{Y}_j = \ln\left\{\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}\right\}$	Estimated Values with Fuzzy Approach: $\widetilde{W}_j(x)$		$\exp\left[\widetilde{W}_j\left(\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}\right)\right]$
			$f_i^C(x)$	$f_i^S(x)$	
1	0.395	-0.928	-1.98	5.38	0.80
2	0.166	-1.794	-1.52	5.93	1.00
.	.	.	.	.	.
89	1.404	0.339	-0.58	3.83	0.76
90	0.030	-3.511	-5.03	4.40	0.65
Total					80.64
MDM					0.896

The $\exp\left[\widetilde{W}_j\left(\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}\right)\right]$ value calculated in Table 5 is the estimated membership degree values of fuzzy numbers in the symmetrical triangular property structure. That is, $\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}$ are the clarified absolute values of observed odds values. In *Table 5*, by substituting the calculated membership degrees of the estimated posibilistic odds values for each cow in (exp $\left[\widetilde{W}_j\left(\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}\right)\right]$)quation 23;

$$\mathrm{MDM} = \frac{1}{90}\sum_{i=1}^{90} 0.80 + 1.0 + \cdots + 0.76 + 0.65 \tag{23}$$

Average membership degree value was calculated as MDM = 0.896. It is seen that this calculated value is greater than the value of the tolerance coefficient (h) of $\exp\left[\widetilde{W}_j\left(\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}\right)\right] \geq h = 0.5$, which means that the assumptions required for valid and reliable results are met [22,27]. At the same time, this calculated value states that the fuzzy logistic regression model in equation 16 is a good model for the analyzed data set. According to this test criterion, the average milk yield values in a lactation period such as milk yield of cows in lactation (X_1), weight (X_2), breeds (X_3), age in lactation (X_4), number of milkings per day (X_5) and milking season (X_6) shows that it can be explained by independent variables with a ratio value of 0.896.

Step 10. Another criterion used to evaluate the goodness of fit of the model is the Mean of Squares Errors (MSE). In order to calculate the MSE value, the estimated fuzzy output values calculated in Table 5 were converted to exact values by clarifying the center of gravity with defuzzification $\mathrm{def\,CoG}\left\{\exp\left[\widetilde{W}_j\left(\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}\right)\right]\right\}$ clarification method in Equation (13).

Thus, the distances between the probability values estimated by clarification and the observed values were calculated. When we want to calculate the $\mathrm{defCoG}\{\exp[\widetilde{W}_i]\}$ value for cow no 1;

$$\mathrm{defCoG}\{\exp[\widetilde{W}_i]\} = \frac{\int_{\exp(-1.98-5.38)}^{\exp(-1.98)} x\left(1-\frac{-1.98-\ln(x)}{5.38}\right)dx + \int_{\exp(-1.98)}^{\exp(-1.98+5.38)} x\left(1-\frac{\ln(x)-(-1.98)}{5.38}\right)dx}{\int_{\exp(-1.98-5.38)}^{\exp(-1.98)} x\left(1-\frac{-1.98-\ln(x)}{5.38}\right)dx + \int_{\exp(-1.98)}^{\exp(-1.98+5.38)} x\left(1-\frac{\ln(x)-(-1.98)}{5.38}\right)dx} \tag{24}$$
$$= 0.0086$$

The values calculated as this are substituted in equation (13), it is calculated as: $\mathrm{defCoG}\{\exp[\widetilde{W}_j]\} = \frac{0.0086+41.7754}{-0.1112+27.3669} = 1.533$. The $\mathrm{defCoG}\{\exp[\widetilde{W}_i]\}$ values in *Table 6* were obtained by calculating the clarified (certain) values of the output values for each other cow in this way.

Table 6. The distance between the predicted possibilistic odds values and the observed values by clarification

No	$\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}$	$\mathrm{defCoG}\{\exp[\widetilde{W}_j]\}$	$\left[\mathrm{defCoG}\{\exp[\widetilde{W}_j]\} - \left(\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}\right)\right]$
1	0.395	1.533	1.138
2	0.166	5.137	4.971
.	.	.	.
89	1.404	3.270	1.866
90	0.030	0.022	-0.008

From each defCoG value calculated in *Table 6*, the observed likelihood (Possibilistic odds) value was subtracted and squared $\left[\mathrm{defCoG}\{\exp[\widetilde{W}_j]\} - \left(\frac{\tilde{\mu}_i}{[1-\tilde{\mu}_i]}\right)\right]^2$. Each squared value was placed in the equation (25) and the MSE value was calculated;

$$\mathrm{MSE} = \frac{1}{90}\sum_{i=1}^{90}[1.533-0.395]^2 + [5.137-0.166]^2 + \cdots + [0.022-0.030]^2 \tag{25}$$

The MSE value calculated by Equation (25) was obtained as 4.871 and $\sqrt{\mathrm{MSE}} = 2.207$. This calculated value is the test criterion value that shows the average distance of the fuzzy values estimated with the fuzzy logistic regression analysis model in Equation (16) from the real observation values.

Discussion

Classical logistic regression analysis method is used to model situations where the values of the dependent variable are categorical. The method can generally be used in educational sciences, social studies, and research related to the diagnosis and treatment of diseases [8,28,29]. However, the use of the method depends on some assumptions. In these studies, reasons such as the expressions of human thoughts and the data sets obtained related to the diagnosis of yield values and animal diseases in livestock, the history of the disease, the unknown environmental factors in the formation of the diseases, and the uncertainty in the symptoms of the disease may cause uncertainties in the diagnostic processes. These assumptions cannot be fulfilled due to the uncertainties arising due to the aforementioned reasons. In these cases, the application of classical statistical methods is not correct [15]. It is a more valid and reliable choice for researchers to use the fuzzy logistic regression analysis method, which is a combination of fuzzy sets and classical statistical theories. It is also a method used in modeling the natural uncertainties in the observation values of the dependent variable or in the relationships. Possibilistic logistic regression analysis method, which is created with the fuzzy logic approach used in different scientific fields, has almost no applications in animal breding in the literature. The application for the estimation of milk yield values of animals belonging to different breeds was discussed for the first time in this study. For fuzzy situations, in the parameter estimations of the fuzzy logistics model, it is thought that the Diamond's Possibilistic method may be a more correct choice to apply to the data obtained for the estimation of yield values in livestock. Doing these and similar studies will gain importance in the solution of all kinds of uncertainties regarding animal husbandry. In the field of veterinary medicine; fuzzy methods were used in various subjects such as estrus detection [30], evaluation of raw milk quality [31], disease diagnosis and determination of risk factors [32]. In their method, fuzzy relation between crisp inputs-crisp output observations is modeled by the proposed model and then compared to the results with a fuzzy neural network method. For some possibilistic methods, one can see the studies done by Taheri and Mirzaei Yeganeh [10], Pourahmad et al.[15], and Atalik and Senturk [8]. Besides, some recent works on distance-based fuzzy logistic regression models are presented by Pourahmad et al.[23], Namdari et al.[28], Salmani et al.[16], and Gao and Lu [17]. Mustafa et al.[9] proposed the fuzzy least square method (FLSM) to determine fuzzy parameters, in that analogue of the conventional normal equations are derived with a suitable metric. Fuzzy logistic regression analysis approach that are presented in these studies are different from other works in the assumptions and optimization method. Pourahmad et al.[15] and Gao and Lu [17] introduced certain least squared fuzzy logistic regression model and evaluated their proposed models by using a capability index for goodness of fit of the model. Namdari et al.[28] used least absolute deviation method to estimate coefficients of a fuzzy logistic regression model and applied measure of performance based on fuzzy distance and a sensitivity index to evaluate the proposed model. Some discussions have been presented offering some modifications on the solution of the above-mentioned exponential possibility regression problems, especially on determining the center of the possibility distribution [3].

To our knowledge, this is the first study of subject "animal husbandry", therefore it can not be compared or discussed with other studies on possibilistic logistic regression in the literature.

In conclusion, it is thought that the approach can be used widely in a short time in studies on animal husbandry and this study can be the basis for similar studies in the future. It can be suggested that researchers who work on this field should consider the possibilistic logistic regression where there is an uncertainity in the dependent variable or in the relationships. It can be concluded that the model can provide the businesses on milk production an efficient and accurate prediction results with minimum deviation by MDM and MSE indices.

Acknowledgements

This research did not receive any specific grant from funding.

References

1. Zadeh LA: Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets Syst*, 100 (1): 9-34, 1999. DOI: 10.1016/S0165-0114(99)80004-9

2. Kim IK, Lee WJ, Yoon JH, Choi SH: Fuzzy regression model using trapezoidal fuzzy numbers for re-auction data. *Int J Fuzzy Log Intell Syst*, 16 (1): 72-80, 2016. DOI: 10.5391/IJFIS.2016.16.1.72

3. Shakouri H, Nadimi GR: A novel fuzzy linear regression model based on a non-equality possibility index and optimum uncertainty. *Appl Soft Comput*, 9 (2): 590-598, 2009. DOI: 10.1016/j.asoc.2008.08.005

4. Namdari M, Yoon JH, Abadi A, Taheri SM, Choi SH: Fuzzy logistic regression with least absolute deviations estimators. *Soft Comput*, 19 (4): 909-917, 2015. DOI: 10.1007/s00500-014-1418-2

5. Chang YHO, Ayyub BM: Fuzyy regression methotds: A comparative assessment. *Fuzzy Set Syst*, 119 (2): 187-203, 2001. DOI: 10.1016/s0165-0114(99)00091-3

6. Sohn SY, Kim DH, Yoon JH: Technology credit scoring model with fuzzy logistic regression. *Appl Soft Comput*, 43, 150-158, 2016. DOI: 10.1016/j.asoc.2016.02.025

7. Tanaka H, Uejima S, Asai K: Linear regression analysis with fuzzy model. *IEEE Trans Syst Man Cybern,* 12 (6): 903-907, 1982.

8. Atalik G, Senturk S: A new approach for parameter estimation in fuzzy regression. *Iran J Fuzzy Syst*, 15 (1): 91-102, 2018. DOI: 10.22111/ijfs.2018.3580

9. Mustafa S, Asghar S, Hanif M: Fuzzy logistic regression based on least square approach and trapezoidal membership function. *Iran J Fuzzy Syst*, 15 (6): 97-106, 2018. DOI: 10.22111/IJFS.2018.4369

10. Taheri SM, Salmani F, Abadi A, Majd HA: A transition model for fuzzy correlated longitudinal responses. *J Intell Fuzzy Syst*, 30 (3): 1265-1273, 2016.

11. Abdalla HA, El-Sayed AA, Hamed R: Fuzzy multinomial logistic regression analysis: A multi-objective programming approach. *AIP Conf Proc*, 1842:030017 2017. DOI: 10.1063/1.4982855

12. Nasrabadi MM, Nasrabadi E: A mathematical-programming approach to fuzzy linear regression analysis. *Appl Math Comput*, 155 (3): 873-881, 2004. DOI: 10.1016/j.amc.2003.07.031

13. Nagar P, Srivastava S: Adaptive fuzzy regression model for the prediction of dichotomous response variables using cancer data: A case study. *J Appl Math Stat Inform*, 4 (2): 183-191, 2008.

14. Namdari M, Abadi A, Taheri S M, Rezaei M, Kalantari M, Omidvar N: Effect of folic acid on appetite in children: Ordinal logistic and fuzzy logistic regressions. *Nutrition*, 30 (3): 274-278, 2014. DOI: 10.1016/j.nut.2013.08.008

15. Pourahmad S, Ayatollahi SMT, Taheri SM: Fuzzy logistic regression: A new possibilistic model and its application in clinical vague status. *Iran J Fuzzy Syst*, 8 (1): 1-17, 2011. DOI: 10.22111/IJFS.2011.232

16. Salmani F, Taheri SM, Yoon JH, Abadi A, Majd HA, Abbaszadeh A: Logistic regression for fuzzy covariates: modeling, inference, and applications. *Int J Fuzzy Syst*, 19 (5): 1635-1644, 2017. DOI: 10.1007/s40815-016-0258-x

17. Gao Y, Lu Q: A fuzzy logistic regression model based on the least squares estimation. *Comp Appl Math*, 37 (3): 3562-3579, 2018. DOI: 10.1007/s40314-017-0531-0

18. Modarres M, Nasrabadi E, Nasrabadi MM: Fuzzy linear regression models with least square errors. *Appl Math Comput*, 163 (2): 977-989, 2005. DOI: 10.1016/j.amc.2004.05.004

19. Ghoshray S: Fuzzy linear regression analysis by symmetric triangular. *Proceedings of IEEE International Conference on Intelligent Engineering Systems*, 307-313, 1997. DOI: 10.1109/INES.1997.632436

20. Tanaka H, Guo P: Possibilistic data analysis for operations research. Studies in Fuzziness and Soft Computing Heidelberg, 194 page, Physica-Verlag, New York. 1999.

21. Kao C, Chyu CL: A fuzzy linear regression model with better explanatory power. *Fuzzy Sets Syst*, 126 (3): 401-409, 2002. DOI: 10.1016/s0165-0114(01)00069-0

22. Khan UT, Valeo C: A new fuzzy linear regression approach for dissolved oxygen prediction. *Hydrolog Sci J*, 60 (6): 1096-1119, 2015. DOI: 10.1080/02626667.2014.900558

23. Pourahmad S, Ayatollahi SMT, Taheri SM, Agahi ZH: Fuzzy logistic regression based on the least squares approach with application in clinical studies. *Comput Math Appl*, 62 (9): 3353-3365, 2011. DOI: 10.1016/j.camwa.2011.08.050

24. Yen KK, Ghoshray S, Roig G: Linear regression model using triangular fuzzy number coefficients. *Fuzzy Sets Syst*, 106 (2): 167-177, 1999. DOI: 10.1016/s0165-0114(97)00269-8

25. Chen LH, Chang CJ: Approaches to select suitable subset of explanatory variables for establishing fuzzy regression models. *J Intell Fuzzy Syst*, *34* (1): 437-457, 2018. DOI: 10.3233/JIFS-17206

26. Hosseinzadeh E, Hassanpour H, Arefi M, Aman M: A weighted goal programming approach to fuzzy linear regression with quasi type-2 fuzzy input-output data. *TWMS J App Eng Math,* 6 (2): 193-212, 2016. DOI: 10.1142/S0217595909002420

27. Tanak HA, Watada J: Possibilistic linear systems and their application to the linear regression model. *Fuzzy Sets Syst,*27 (3): 275-289, 1988. DOI: 10.1016/0165-0114(88)90054-1

28. Namdari M, Taheri SM, Abadi A, Rezaei M, Kalantari N: Possibilistic logistic regression for fuzzy categorical response data. *2013 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, 1-6, 2013. DOI: 10.1109/FUZZ-IEEE.2013.6622457

29. Tanaka H: Fuzzy data analysis by possibilistic linear models. *Fuzzy Sets Syst*, 24 (3): 363-375, 1987. DOI: 10.1016/0165-0114(87)90033-9

30. Memmedova N, Keskin İ: Oestrus detection by fuzzy logic model using trait activity in cows. *Kafkas Univ Vet Fak Derg*, 17 (6): 1003-1008, 2011. DOI: 10.9775/kvfd.2011.4960

31. Akıllı A, Atıl H, Takma Ç, Ayyılmaz T: Fuzzy logic-based decision support system for dairy cattle. *Kafkas Univ Vet Fak Derg*, 22 (1): 13-19, 2016. DOI: 10.9775/kvfd.2015.13516

32. Cihan P, Gökçe E, Kalıpsız O: A review of machine learning applications in veterinary field. *Kafkas Univ Vet Fak Derg*, 23 (4): 673-680, 2017. DOI: 10.9775/kvfd.2016.172816